02ELMA - Homework 8

Assigned for the week of Apr 7, 2025

Questions

1. A circular wire loop, with radius R, lies in the xy plane, centered at the origin, and carries a current I running counterclockwise as viewed from the positive z axis.

   (a) What is its magnetic dipole moment?

   (b) What is the approximate magnetic field at points far from the origin?

   (c) Recall the exact magnetic field expression derived for a circular loop for the z axis by using the Biot-Savart law. Show that, for points on the z axis, the answer you found for (b) is consistent with the exact field when $z \gg R$.

2. Consider Rutherford's atomic theory: A positive charge ($+e$) is located in the center, around which a negative charge ($-e$) orbits in a circular motion with radius R and linear velocity v. Although technically this orbital motion does not constitute a steady current, in practice the period of motion is so short that it can be approximated as a steady current. If the negative charge rotates counterclockwise, as observed from the positive z axis, what is the magnitude and direction of the orbital magnetic dipole moment in terms of e, v, and R?

3. Consider a uniformly magnetized sphere with magnetization $\vec{M} = M\hat{z}$ and radius R.

   (a) Calculate volume and surface bound currents, $\vec{J}_b$ and $\vec{K}_b$. (Hint: Consider the spherical coordinates while calculating the cross products)

   (b) Think of a spherical shell with radius R rotating about z axis with constant angular velocity ω. Note that the linear velocity of a point on the surface $\vec{v}$ must be a function of θ in spherical coordinates (as θ increases, the point on the sphere covers more distance per unit of time). Express $\vec{v}$ in terms of ω, R, θ (don't forget its direction).

   (c) If the spherical shell in (b) has a surface charge density σ, find its surface current density $\vec{K}$.

   (d) Compare $\vec{K}_b$, the bound surface current obtained for a uniformly magnetized sphere, and K, the surface current density of a rotating spherical shell with angular velocity ω. Explain the analogy between these two cases. What is the relationship between M and σ, ω and R?

   (e) Write down the magnetic dipole moment $\vec{m}$ of the magnetized sphere.

4. Utilize the insights gained from the preceding question regarding the magnetized sphere and the rotating spherical shell with a surface charge density to determine the equatorial linear velocity of an electron (assuming that electron is a rotating spherical shell). Refer to the classical radius for the electron as found in Homework 4. Compare your result with the speed of light.

Given for 1: The magnetic field created by a pure magnetic dipole:

$$\vec{B}_{dip}(r, \theta) = \frac{\mu_0 m}{4\pi r^3}\left[2\cos(\theta)\hat{r} + \sin(\theta)\hat{\theta}\right]$$