02YMECH - Homework 1

Assigned for the week of Sep 22, 2025

Questions

1. Compute following indefinite integrals

$$\int \frac{dx}{a+x} \text{ and } \int \left(3x^2\, e^{x^3} + \frac{4x}{1+x^2}\right) dx,$$

   where a is a constant.

2. Compute the definite integral

$$\int_1^4 (2x+3)\, dx.$$

3. Find the derivative of

$$f(x) = \frac{x^3 \ln(x)}{e^{2x}}, \qquad (x > 0).$$

4. Find the local minimum and maximum values of the function

$$f(x) = x^3 - 6x^2 + 9x + 1$$

   by using the first and second derivative tests.

5. Find the Taylor series of $\sin x$, $\cos x$, and e^x about $x = 0$ up to and including the term in x^5.

6. Compute the partial derivatives of the function

$$f(x, y) = x^2 y + e^{xy}$$

   with respect to x and y and write down the total differential df.

7. The position of a particle is described by the following vector as a function of time t:

$$\mathbf{r}(t) = a\cos(t)\,\hat{\imath} + a\sin(t)\,\hat{\jmath} + ct\,\hat{k}$$

   where a and c are constants with units of length and velocity, respectively.

   (a) What kind of trajectory does the particle follow?

   (b) Find the velocity, acceleration and speed of the particle as functions of time.

(c) When you consider the trajectory of the particle again, are the directions of the velocity and acceleration vectors in the way you expect? Explain.

(d) Calculate the projection of the acceleration vector onto the velocity vector as a function of time and give a physical interpretation of this result.

8. A flat rectangular surface lies in the xy-plane. The surface has sides of length $a = 0.20$ m along the x-axis and $b = 0.30$ m along the y-axis. A uniform water flow is described by the flux vector field:

$$\mathbf{F} = 200\,\hat{\imath} + 300\,\hat{\jmath} + 400\,\hat{k} \quad \left[\frac{\mathrm{L}}{\mathrm{m}^2 \cdot \mathrm{s}}\right]$$

(a) Compute the area vector $\mathbf{A}$ of the surface (the vector normal to the surface with magnitude equal to the area of the surface).

(b) Compute the volume of water per unit time (in liters per second) passing through the surface using:

$$Q = \mathbf{F} \cdot \mathbf{A}.$$

(c) Discuss how the orientation of the surface affects the water flow through it.

9. A particle moves in a plane such that its position vector at time t is given by

$$\mathbf{r}(t) = at^2\,\hat{\imath} + bt^3\,\hat{\jmath}.$$

where a and b are constants with appropriate units.

(a) Find the velocity and acceleration vectors as functions of time.

(b) Find the angle between the velocity and acceleration vectors at time $t = 1$.

10. Given the vectors

$$\mathbf{A} = 2\,\hat{\imath} - 3\,\hat{\jmath} + \hat{k}, \quad \mathbf{B} = -\hat{\imath} + 4\,\hat{\jmath} + 2\,\hat{k},$$

compute $\mathbf{A} \times \mathbf{B}$.