

02YMECH - Homework 4

Assigned for the week of Oct 13, 2025

Questions

1. The position of a particle hooked to a spring is given by the equation $x(t) = A \cos(\omega_0 t + \phi)$, where A , ω_0 , and ϕ are constants. If the particle's initial position is x_0 and initial velocity is v_0 , how long will it take for the particle to reach its maximum displacement for the first time?
2. What is the average kinetic energy and average potential energy of a harmonic oscillator over one complete cycle? Express your answers in terms of the total energy E of the system. (Consider the time integrals of kinetic and potential energy over one period.)
3. A simple pendulum consists of a bob of mass m attached to a light, inextensible string of length L , staying in its equilibrium position. The support point of the pendulum is suddenly accelerated horizontally with constant acceleration a (e.g., the ceiling begins to move to the right).
 - (a) Determine the new equilibrium angle θ that the pendulum makes with the vertical.
 - (b) Find the period of small oscillations about this new equilibrium position.
4. The position of a damped harmonic oscillator is given by the equation $x(t) = Ae^{-\beta t} \cos(\omega_1 t + \phi)$, where A , β , ω_1 , and ϕ are constants. How much time will it take for the amplitude of the oscillations to decrease to half of its initial value?